

## Multiplication

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads, they use an efficient written method accurately and with confidence.

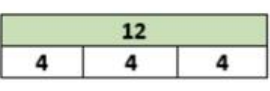
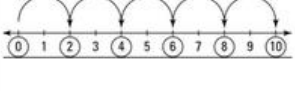
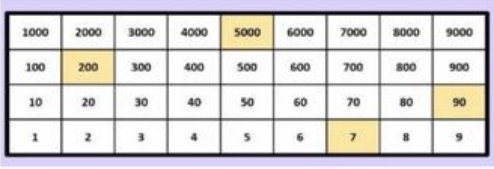
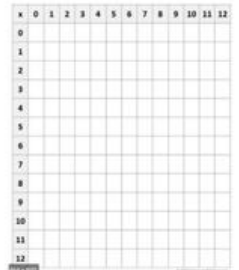
To multiply successfully, children need to be able to:

- Count forwards and backwards in 2's, 5's and 10's
- Double numbers to 10
- Add two or more single digit numbers mentally
- Recognise multiplication as repeated addition

## Vocabulary for Multiplication

Equal  
Groups  
Times  
Multiply  
Multiplication  
Factor  
Product  
Array - row, column  
Double  
Repeated addition

## Models

|                                                                                                              |                                                                                                     |                                                                                                                                                                                                                                    |                                                                                                              |
|--------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------|
| <b>Grouping</b><br>       | <b>Array</b><br> | <b>Bar Models</b><br>                                                                                                                          | <b>Number Lines</b><br> |
| <b>Gattegno Chart</b><br> |                                                                                                     | <b>12x12 multiplication table with zero included.</b><br><br><b>In Year 2 Children will complete only 0x, 1x, 2x, 5x, 10x of the grid</b><br> |                                                                                                              |

## Manipulatives:

|                                                                                                                    |                                                                                                        |                                                                                                                                                |
|--------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Natural objects/toys</b><br> | <b>Counters</b><br> | <b>Counters to represent a group, marked 2, 5, 10</b><br> |
|--------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------|

## Learning Pathway for Multiplication

Sorting & noticing groups

Making a range of groups for example equal groups, unequal groups, large/small groups and noticing what is the same/different, what does/doesn't belong in each group.  
Objects, colours, amounts etc.



Making pairs

Matching pairs - socks, gloves etc.



Explore stories and songs of 2's

For example Noah's ark story & song

Skip counting in 2's

Children use a variety of representations and models appropriate to the child's stage of learning to learn to skip count in twos (even numbers only) both forwards and backwards.



|      |      |      |      |      |      |      |      |      |
|------|------|------|------|------|------|------|------|------|
| 1000 | 2000 | 3000 | 4000 | 5000 | 6000 | 7000 | 8000 | 9000 |
| 100  | 200  | 300  | 400  | 500  | 600  | 700  | 800  | 900  |
| 10   | 20   | 30   | 40   | 50   | 60   | 70   | 80   | 90   |
| 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    |

Counting in twos

(Then repeated for 5's and 10's)

Natural pairs

Children begin by counting the number of groups/pairs.

One group of two, two groups of two, three groups of two etc.

This can then be abbreviated.

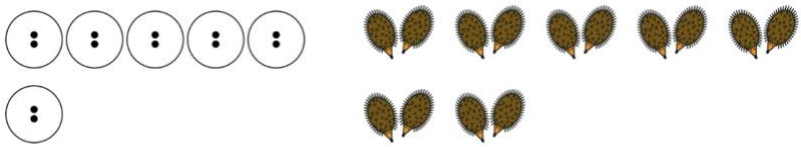
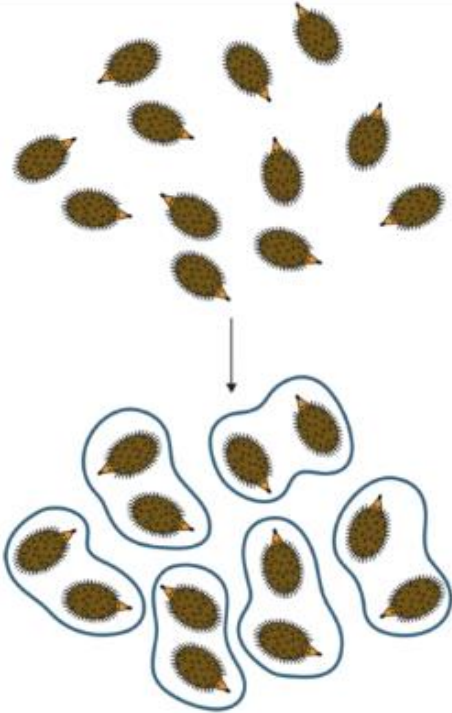
One two, two twos, three twos etc.

Children then count the objects in the groups (wheels).

2,4,6,8,10,12,14,16



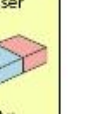
Pre money tokens and objects grouped into 2's.

|                                                                                   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
|-----------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                                                                   | <p>The same approach should then be used to count pre money tokens and objects that have been grouped into 2's.</p>                                                                                                                                                                                                                                                                                                                                                      |
| <p>The efficiency of counting in twos</p> <p>(Then repeated for 5's and 10's)</p> | <p>Children learn that counting in 2's can be more efficient than counting in 1's.</p>  <p>Begin by counting an even number of physical objects in 1's. Then count the same objects again but counting in groups of 2 (pairing off). Discuss which was the quickest/easiest/most efficient way of finding the total.</p> <p>Next, progress to counting pictorial representations and encourage children to draw rings to create groups of 2 before counting in 2's.</p> |
| Counting in tens                                                                  | <p>The same process should be repeated as with counting in 2's above (skip counting in tens, counting in natural tens, the efficiency of counting in ten).</p> <p>Fingers and thumbs on hands should be used as natural groups of ten.</p>                                                                                                                                                                                                                                                                                                                 |
| Counting in fives                                                                 | <p>The same process should be repeated as with counting in 2's above (skip counting in fives, counting in natural fives, the efficiency of counting in five).</p> <p>Fingers and thumbs on hands should be used as natural groups of five.</p>                                                                                                                                                                                                                                                                                                             |
| Know coins have different values                                                  | <p>Children will need to understand that coins have a value which is not dependent on its size, colour or shape.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                       |

| Coin | Heads                                                                             | Tails                                                                             | Pre-money token                                                                    |
|------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|------------------------------------------------------------------------------------|
| 1p   |  |  |  |
| 2p   |  |  |  |
| 5p   |  |  |  |
| 10p  |  |  |  |

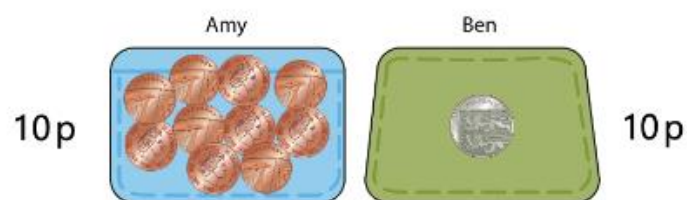
Understand a single coin can be worth the same as several pennies

The focus here is on comparing a single coin to the **equivalent in pennies** NOT for example 10p being equal to two 5p coins.

|                                                                                                     |                                                                                                     |                                                                                                                 |                                                                                                        |
|-----------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------|
| Pencil<br><br>6p | Marble<br><br>4p | Mini spinning top<br><br>9p | Eraser<br><br>10p |
|-----------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------|

Can Amy buy an eraser? Yes ✓

Can Ben buy an eraser? Yes ✓



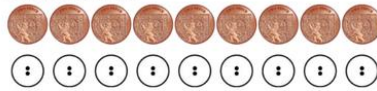
Stem Sentences:

I say 5p but I think 5 pennies

I say 10p but i think 10 pennies

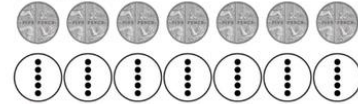
Find the value of a set of 2p, 5p, 10p coins using skip counting skills.

Finding the value of a set of 2 p coins – coins and pre-money tokens:



- 'One two-pennies, two two-pennies, three two-pennies...'
- 'Two p, four p, six p...'
- 'There are nine coins.'
- 'Each coin has a value of two p.'
- 'This is eighteen p.'

Finding the value of a set of 5 p coins – coins and pre-money tokens:



- 'One five-pennies, two five-pennies, three five-pennies...'
- 'Five p, ten p, fifteen p...'
- 'There are seven coins.'
- 'Each coin has a value of five p.'
- 'This is thirty-five p.'

Finding the value of a set of 10 p coins – coins and pre-money tokens:



- 'One ten-pennies, two ten-pennies, three ten-pennies...'
- 'Ten p, twenty p, thirty p...'
- 'There are nine coins.'
- 'Each coin has a value of ten p.'
- 'This is ninety p.'

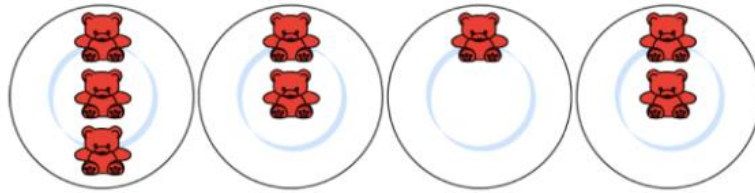
Equal groups using pictorial representations

Children distinguish between equal and unequal groups on pictorial representations.

|  | Equal groups | Unequal groups |
|--|--------------|----------------|
|  |              |                |
|  |              |                |
|  |              |                |
|  |              |                |
|  |              |                |

Next, children use concrete objects to make unequal groups equal.

*'Make these groups equal.'*



Children describe equal groups



There are five equal groups of cakes.  
There are three cakes in each group.  
There are five groups of three.

Understand equal groups as repeated addition

Children apply their knowledge of groups to repeated addition.

There are four groups of two.  
There are two and two and two and two.  
We can write this as two plus two plus two plus two.  
The two represents the number of children at the desk.  
There are four twos because there are four groups of two.



|   |   |   |   |
|---|---|---|---|
| 2 | 2 | 2 | 2 |
|---|---|---|---|

2 + 2 + 2 + 2

Children will then complete an addition expression to describe a pictorial representation.

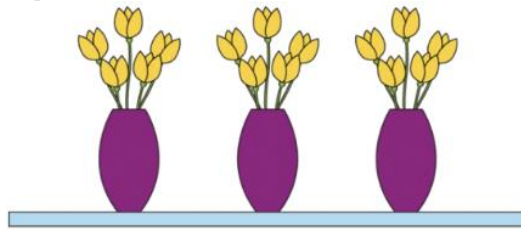


+  +



Equal groups can be represented as a multiplication expression

Children apply their knowledge of equal groups to multiplication expressions.



There are three groups of five.

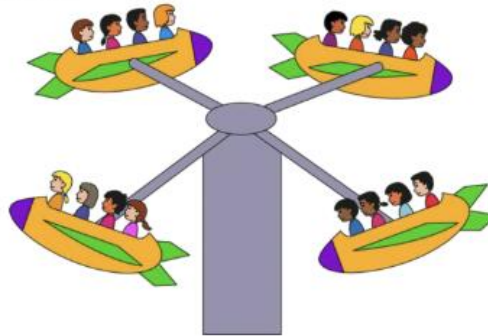
There are three fives.

We can write this as three times five.

(note that the equals sign is not introduced at this stage)

Children will then complete

*'Write a multiplication expression to match the picture.'*



Multiplication expressions to describe a pictorial representation.

Multiplication expressions where the groups contain zero or one item

### Group size = 0



- 'There are three groups of zero eggs.'
  - 'We can write this as zero plus zero plus zero.'
- $$0 + 0 + 0$$
- 'We can write this as three times zero.'
- $$3 \times 0$$

### Group size = 1



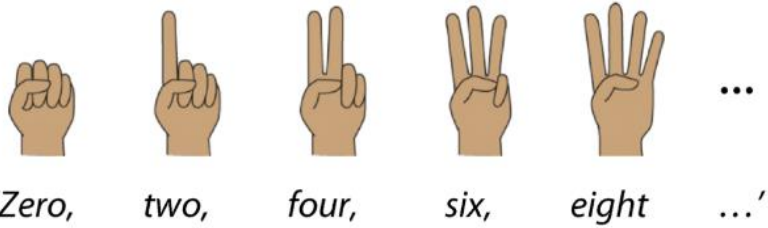
'There are four groups of one football.'

|   |   |   |   |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
|---|---|---|---|

- 'We can write this as one plus one plus one plus one.'
- $$1 + 1 + 1 + 1$$
- 'We can write this as four times one.'
- $$4 \times 1$$

Silent tally in 2's 5's 10's

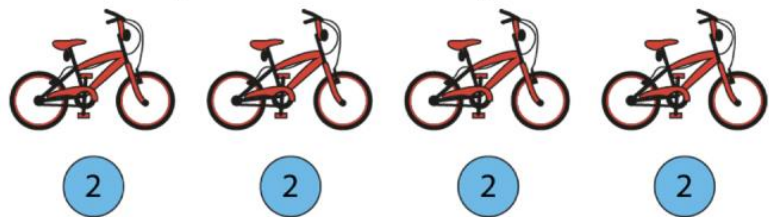
Children learn to skip count using a 'silent tally' putting up each finger as they count in a group of 2, 5 or 10 understanding that when they have 3 fingers up this means there are 3 groups of 2, 5 or 10.



Understanding and identifying the 'factors' and 'product'.

(Taught through 2x times table)

Introduce the vocabulary 'factor' to represent the number of groups and the amount in



- 'Two, four, six, eight.'
- 'There are eight wheels.'
- $4 \times 2 = 8$        $8 = 4 \times 2$
- 'Four is a factor.'
- 'Two is a factor.'
- 'The product of four and two is eight.'
- 'Eight is the product of four and two.'

each group. Children learn that a multiplication expression can be turned into an equation (introduce equal sign and product).

Know that multiplication is commutative.

(Taught through 2x times table)

It is important that children understand that the factors and product can be reversed.

|        |   |        |   |         |
|--------|---|--------|---|---------|
| 3      | × | 2      | = | 6       |
| factor | × | factor | = | product |

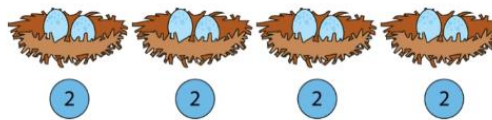
|         |   |        |   |        |
|---------|---|--------|---|--------|
| 6       | = | 3      | × | 2      |
| product | = | factor | × | factor |



Know that factor pairs can be written any order and the product remains the same. (Commutative)

(Taught through 2x times table)

Children will now explore real world representations using the models and manipulatives explored so



$$2 + 2 + 2 + 2 = 8$$

- 'There are four groups of two eggs. There are eight eggs altogether.'

$$4 \times 2 = 8$$

- 'There are two eggs, four times. There are eight eggs altogether.'

$$2 \times 4 = 8$$

far to discover that the factors can be written in any order, including money examples.



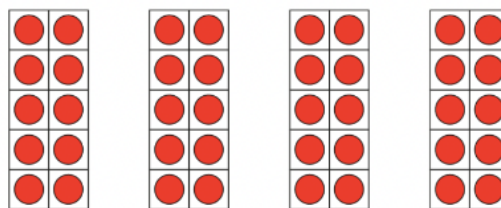
$$\boxed{9} \times \boxed{2} = \boxed{18}$$

$$\boxed{2} \times \boxed{9} = \boxed{18}$$

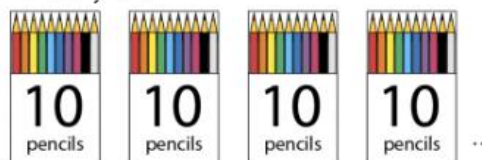
Know counting in tens can be represented by the ten times table.

Recognise and use facts from the tens times table to solve problems.

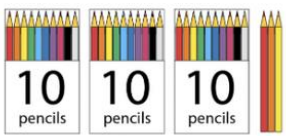
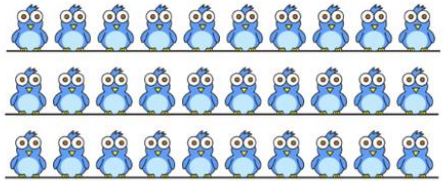
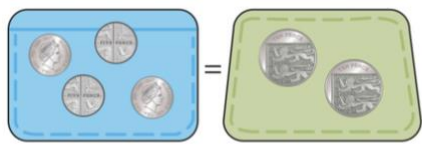
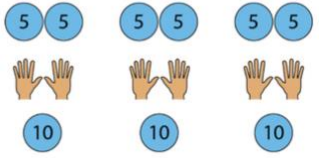
Children will review counting in tens to begin with where the cardinality can be seen such as on tens frames.

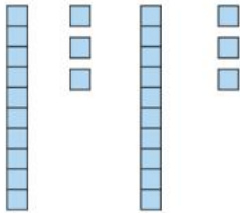
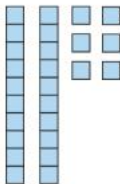


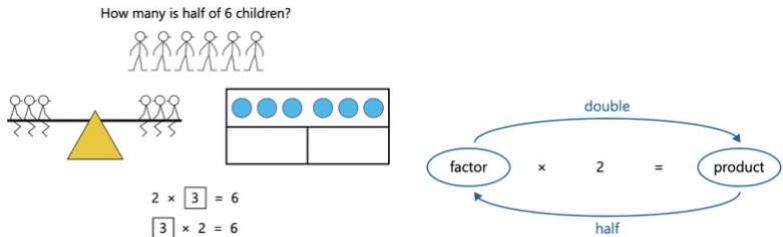
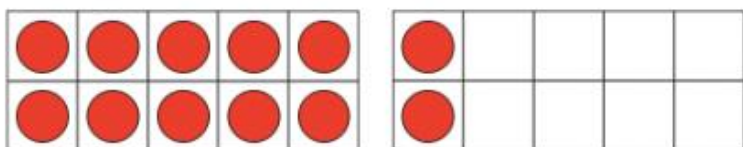
Then move on to counting in tens where the cardinality cannot be seen for example coins representing a value of 10p or a bucket labelled with 10 litres.



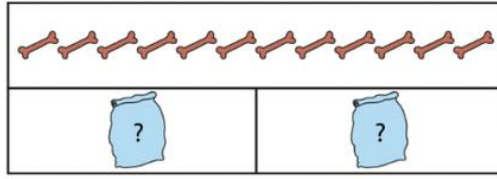
Finally apply knowledge to solve a range of problems.

|                                                            |                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                                            | <p>Which expression(s) match this picture?</p>  <p>There are 10 birds on each wire.<br/>Altogether there are 30 birds. How many wires are there?</p>  <p> <math>4 \times 10</math><br/> <math>4 \times 3</math><br/> <math>10 \times 3 + 3</math><br/> <math>3 \times 10 + 3</math> </p> |
| Repeat the previous step using the 5 times table.          |  <p>5 5 5</p> <p>5 10 15<br/>five ten fifteen</p> <p><math>3 \times 5 = 15</math></p>                                                                                                                                                                                                                                                                                     |
| Explore the relationship between multiples of 10's and 5's |  <p>Amelia = Freddie</p> <p><math>4 \times 5p</math> <math>2 \times 10p</math></p>  <p>5 5 5 5 5 5</p> <p>10 10 10</p> <p><math>6 \times 5 = 30</math><br/><math>3 \times 10 = 30</math></p>                                                                                        |
| Double two digit numbers (max total of 100)                | <p>The children will use dienes manipulatives to explore the doubling of two digit numbers.</p> <p>First making the amount twice (step 1 below). Then grouping the tens and ones before using addition skills and knowledge to find the sum of the tens and the sum of the ones. Finally children add those to find the product. (step 2 below)</p>                                                                                                         |

|                                                                            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |   |  |   |   |
|----------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|--|---|---|
|                                                                            | <p>double 13</p>  <p>Step 1      double 10 + double 3</p> <p>double 13 = 26</p>  <p>double 10 + double 3</p> <p>Step 2      20 + 6 = 26</p>                                                                                                                                                                                                                                      |   |  |   |   |
| <p>Know that multiplication with a factor of 2 is the same as doubling</p> | <p>Children will explore in more depth that 2 equal groups means a double and so multiplication equations with a factor of 2 is a double.</p> <p>There were 3 birds. Now there are double that number.<br/>How many birds now?</p>  <p>Double 3 is the same as 2 groups of 3.</p> <table border="1" data-bbox="908 1565 1295 1713"> <tr> <td colspan="2">6</td></tr> <tr> <td>3</td><td>3</td></tr> </table> <p><math>2 \times 3 = 6</math><br/>2 times 3 = 6</p> | 6 |  | 3 | 3 |
| 6                                                                          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |   |  |   |   |
| 3                                                                          | 3                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |  |   |   |
| <p>Know that halving is the inverse of doubling</p>                        | <p>Children will explore practically the relationship between doubling and halving and then begin to use this knowledge to help them solve missing number multiplication equations.</p>                                                                                                                                                                                                                                                                                                                                                              |   |  |   |   |

|                                                                                   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |   |  |   |   |
|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|--|---|---|
|                                                                                   | <p>How many is half of 6 children?</p>  <p><math>2 \times \boxed{3} = 6</math><br/><math>\boxed{3} \times 2 = 6</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |   |  |   |   |
| Halve two digit numbers (to 20) to find missing parts of multiplication equations | <p><b>Method 1:</b><br/>Apply knowledge of doubles to find one half</p> <p style="text-align: center;"><b>half of 8 = 4</b></p> <table border="1" style="margin: 10px auto; width: 250px; height: 80px;"><tr><td colspan="2" style="text-align: center; padding: 10px;">8</td></tr><tr><td style="text-align: center; padding: 10px;">4</td><td style="text-align: center; padding: 10px;">4</td></tr></table> <p>Double <span style="border: 1px solid black; padding: 0 5px;">4</span> is equal to 8; so half of 8 is equal to <span style="border: 1px solid black; padding: 0 5px;">4</span> .</p> <p><b>Method 2:</b> When halving even teens numbers.<br/>Children will use tens frames and counters to support their understanding. First find half of the ten, then find half of the ones and finally add these together.</p> <p style="text-align: center;"><b>half of 12 = 6</b></p> <div style="text-align: center; margin: 10px 0;"></div> <p style="text-align: center;"><b>half of 10 + half of 2</b></p> <p style="text-align: center; font-size: 1.2em;">5      +      1      =      6</p> <p><b>Method 3: Cardinality visible (can see each individual item represented)</b><br/>Know and use the stem sentence : “When one of the factors is 2. The other factor is half of the product.”</p> | 8 |  | 4 | 4 |
| 8                                                                                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |   |  |   |   |
| 4                                                                                 | 4                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |   |  |   |   |

*'There are two packets of dog bones. Each packet has the same number of bones. There are twelve bones altogether. How many bones are there in each packet?'*



$$2 \times \square = 12 \quad \square \times 2 = 12$$

|    |   |
|----|---|
| 12 |   |
| ?  | ? |

- 'One of the factors is two, so the other factor is half of the product.'
- 'Half of the bones are in each packet.'
- 'I know that half of twelve is equal to six.'
- 'So there are six dog bones in each packet.'

**Method 4: Cardinality not visible (one item represents more than 1 e.g money)**

*'Emily has £20 in total, kept in two money-boxes. Each money-box contains the same amount of money. How much is in each money-box?'*



$$2 \times \square = 20 \quad \square \times 2 = 20$$

|    |   |
|----|---|
| 20 |   |
| ?  | ? |

- 'One of the factors is two, so the other factor is half of the product.'
- 'Half of the money is in each money-box.'
- 'Half of two tens is equal to one ten.'
- 'So there is ten pounds in each money-box.'